

Package ‘mrf’

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Type Package

Title Multiresolution Forecasting

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Description

Forecasting of univariate time series using feature extraction with variable prediction methods is provided. Feature extraction is done with a redundant Haar wavelet transform with filter $h = (0.5, 0.5)$. The advantage of the approach compared to typical Fourier based methods is an dynamic adaptation to varying seasonalities. Currently implemented prediction methods based on the selected wavelets levels and scales are a regression and a multi-layer perceptron. Forecasts can be computed for horizon 1 or higher. Model selection is performed with an evolutionary optimization. Selection criteria are currently the AIC criterion, the Mean Absolute Error or the Mean Root Error. The data is split into three parts for model selection: Training, test, and evaluation dataset. The training data is for computing the weights of a parameter set. The test data is for choosing the best parameter set. The evaluation data is for assessing the forecast performance of the best parameter set on new data unknown to the model. This work is published in Stier, Q.; Gehlert, T.; Thrun, M.C. Multiresolution Forecasting for Industrial Applications. Processes 2021, 9, 1697. <doi:10.3390/pr9101697>.

Imports limSolve, DEoptim, stats, forecast, monmlp, nnfor

Suggests knitr, rmarkdown

Depends R (>= 3.5.0)

License GPL-3

Encoding UTF-8

LazyData true

VignetteBuilder knitr

URL <https://www.deepbionics.org>

BugReports <https://github.com/Quirinms/MRFR/issues>

NeedsCompilation no

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mrf-package	<i>Multiresolution Forecasting</i>
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Description

Forecasting of univariate time series using feature extraction with variable prediction methods is provided. Feature extraction is done with a redundant Haar wavelet transform with filter $h = (0.5, 0.5)$. The advantage of the approach compared to typical Fourier based methods is an dynamic adaptation to varying seasonalities. Currently implemented prediction methods based on the selected wavelets levels and scales are a regression and a multi-layer perceptron. Forecasts can be computed for horizon 1 or higher. Model selection is performed with an evolutionary optimization. Selection criteria are currently the AIC criterion, the Mean Absolute Error or the Mean Root Error. The data is split into three parts for model selection: Training, test, and evaluation dataset. The training data is for computing the weights of a parameter set. The test data is for choosing the best parameter set. The evaluation data is for assessing the forecast performance of the best parameter set on new data unknown to the model. This work is published in Stier, Q.; Gehlert, T.; Thrun, M.C. Multiresolution Forecasting for Industrial Applications. Processes 2021, 9, 1697. <doi:10.3390/pr9101697>. The package consists of a multiresolution forecasting method using a

redundant Haar wavelet transform based on the manuscript [Stier et al., 2021] which is currently in press. One-step and multi-step forecasts are computable with this method. Nested and non-nested cross validation is possible.

Details

Forecasting of univariate time series using feature extraction with variable prediction methods is provided. Feature extraction is done with a redundant Haar wavelet transform with filter $h = (0.5, 0.5)$. The advantage of the approach compared to typical Fourier based methods is an dynamic adaptation to varying seasonalities. Currently implemented prediction methods based on the selected wavelets levels and scales are a regression and a multi-layer perceptron. Forecasts can be computed for horizon 1 or higher. Model selection is performed with an evolutionary optimization. Selection criterias are currently the AIC criterion, the Mean Absolute Error or the Mean Root Error. The data is split into three parts for model selection: Training, test, and evaluation dataset. The training data is for computing the weights of a parameter set. The test data is for choosing the best parameter set. The evaluation data is for assessing the forecast performance of the best parameter set on new data unknown to the model.

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Author(s)

Quirin Stier

References

[Stier et al., 2021] Stier, Q.; Gehlert, T.; Thrun, M.C. Multiresolution Forecasting for Industrial Applications. *Processes* 2021, 9, 1697. <https://doi.org/10.3390/pr9101697>

entsoe

Entsoe DataFrame containing Time Series

Description

Data from a European Network of Transmission System Operators for Electricity Accessed: 2020-08-20, 2019. Time series contains 3652 data points without missing values. Data describes electric load for time range between 2006 and 2015

Usage

`data(entsoe)`

Format

A DataFrame with 3652 rows and 2 columns

Source

[Archive](#)

Examples

```
data(entsoe)
data = entsoe$value
```

mrf_elm_forecast	<i>Forecast with Extreme Learning Machines</i>
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Description

This function creates a one step forecast using a multi layer perceptron with one hidden Layer. The number of input is the sum of all coefficients chosen with the parameter CoefficientCombination. The CoefficientCombination parameter controls the number of coefficients chosen for each wavelet and smooth part level individually.

Usage

```
mrf_elm_forecast(UnivariateData, Horizon, Aggregation, Threshold="hard",
Lambda=0.05)
```

Arguments

UnivariateData	[1:n] Numerical vector with n values.
Horizon	Number indicating horizon for forecast from 1 to horizon.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series.
Threshold	Character indicating if Thresholding is done on the wavelet decomposition or not. Default: Threshold="hard". Possible entries: Threshold="hard" for hard thresholding. Threshold="soft" for soft thresholding. Any other input indicates no thresholding.
Lambda	Numeric value indicating the threshold for computing a hard or soft threshold on the wavelet decomposition.

Value

forecast	Numerical value with one step forecast
----------	--

Author(s)

Quirin Stier

References

Aussem, A., Campbell, J., and Murtagh, F. Waveletbased Feature Extraction and Decomposition Strategies for Financial Forecasting. *International Journal of Computational Intelligence in Finance*, 6,5-12, 1998.

Renaud, O., Starck, J.-L., and Murtagh, F. Prediction based on a Multiscale De- composition. *International Journal of Wavelets, Multiresolution and Information Processing*, 1(2):217-232. doi:10.1142/S0219691303000153, 2003.

Murtagh, F., Starck, J.-L., and Renaud, O. On Neuro-Wavelet Modeling. *Decision Support Systems*, 37(4):475-484. doi:10.1016/S0167-9236(03)00092-7, 2004.

Renaud, O., Starck, J.-L., and Murtagh, F. Wavelet-based combined Signal Filter- ing and Prediction. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 35(6):1241-1251. doi:10.1109/TSMCB.2005.850182, 2005.

Examples

```
data(AirPassengers)
len_data = length(as.vector(array(AirPassengers)))
UnivariateData = as.vector(AirPassengers)[1:(len_data-1)]
Aggregation = c(2,4)
if(requireNamespace('nnfor', quietly = TRUE)){
  forecast = mrf_elm_forecast(UnivariateData, Horizon=1, Aggregation)
  true_value = array(AirPassengers)[len_data]
  error = true_value - forecast
}
```

mrf_forecast

Multiresolution Forecast

Description

Creates a multiresolution forecast for a given multiresolution model based on [Stier et al., 2021] which is currently in press. (mrf_train).

Usage

```
mrf_forecast(Model, Horizon=1)
```

Arguments

Model	List containing model specifications from mrf_train().
Horizon	Number indicating horizon for forecast from 1 to horizon.

Value

List of

Forecast [1:Horizon] Numerical vector with forecast of horizon according to its index.
 Model List containing model specifications from mrf_train().

Author(s)

Quirin Stier

References

[Stier et al., 2021] Stier, Q., Gehlert, T. and Thrun, M. C.: Multiresolution Forecasting for Industrial Applications, *Processess*, 2021.

Examples

```
data(AirPassengers)
Data = as.vector(AirPassengers)
len_data = length(Data)
Train = Data[1:(len_data-2)]
Test = Data[(len_data-1):len_data]
# One-step forecast (Multiresolution Forecast)
model = mrf_train(Train)
one_step = mrf_forecast(model, Horizon=1)
Error = one_step$Forecast - Test[1]
# Multi-step forecast (Multiresolution Forecast)
# Horizon = 2 => Forecast with Horizon 1 and 2 as vector
model = mrf_train(Train, Horizon=2)
multi_step = mrf_forecast(model, Horizon=2)
Error = multi_step$Forecast - Test
```

mrf_model_selection *Model selection for Multiresolution Forecasts*

Description

Evaluates the best coefficient combination for a given aggregation scheme based on a rolling forecasting origin based on the manuscript [Stier et al., 2021] which is currently in press.

Usage

```
mrf_model_selection(UnivariateData, Aggregation, Horizon = 1, Window = 2,
  Method = "r", crit = "AIC", itermax = 1, lower_limit = 1, upper_limit = 2,
  NumClusters = 1, Threshold="hard", Lambda=0.05)
```

Arguments

UnivariateData	[1:n] Numerical vector with n values.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series.
Horizon	Number indicating horizon for forecast from 1 to horizon.
Window	Number indicating how many points are used for cross validation.
Method	String indicating which method to use. Available methods: 'r' = Autoregression. 'nn' = Neural Network. 'elm' = Extreme Learning Machine. 'nnetar' = forecast::nnetar. Default: Method="r".
crit	String with criterion. Available criterions: "AIC" = Akaikes Info. Crit. "MAE" = Mean Abs. Error. "MRE" = Mean Root Error. Default: crit = "AIC".
itermax	Number of iterations used in the differential evolutionary optimization algorithm. Default: itermax = 1.
lower_limit	[1:Scales+1] Numeric vector: Lower limit for coefficients selected for each level.
upper_limit	[1:Scales+1] Numeric vector: Higher limit for coefficients selected for each level.
NumClusters	Number of clusters used for parallel computing. Default: NumClusters = 1.
Threshold	Character indicating if Thresholding is done on the wavelet decomposition or not. Default: Threshold="hard". Possible entries: Threshold="hard" for hard thresholding. Threshold="soft" for soft thresholding. Any other input indicates no thresholding.
Lambda	Numeric value indicating the threshold for computing a hard or soft threshold on the wavelet decomposition.

Details

The evaluation function (optimization function) is built with a rolling forecasting origin (rolling_window function), which computes a h-step ahead forecast (for $h = 1, \dots, \text{horizon}$) for 'Window' many steps. The input space is searched with an evolutionary optimization method. The search is restricted to one fixed aggregation scheme (parameter: 'Aggregation'). The deployed forecast method can be an autoregression or a neural network (multilayer perceptron with one hidden layer).

Value

CoefficientCombination	[1:Scales+1] Numerical vector with numbers which are associated with wavelet levels. The last number is associated with the smooth level. Each number determines the number of coefficient used per level. The selection follows a specific scheme. Best combination of coefficients found by the model selection algorithm.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series. Best Aggregation scheme found by the model selection algorithm.

Author(s)

Quirin Stier

References

[Stier et al., 2021] Stier, Q., Gehlert, T. and Thrun, M. C.: Multiresolution Forecasting for Industrial Applications, Processess, 2021.

Examples

```
data(entsoe)
UnivariateData = entsoe$value
Aggregation = c(2,4)
res = mrf_model_selection(UnivariateData, Aggregation, Horizon = 1, Window = 2,
Method = "r", crit = "AIC", itermax = 1, lower_limit = 1, upper_limit = 2,
NumClusters = 1)
BestCoefficientCombination = res$CoefficientCombination
```

mrf_multi_step_forecast

Multiresolution Forecast

Description

This function creates a multi step forecast for all horizons from 1 to steps based on the manuscript [Stier et al., 2021] which is currently in press. The deployed forecast method can be an autoregression or a neural network (multilayer perceptron with one hidden layer). Multi step forecasts are computed recursively.

Usage

```
mrf_multi_step_forecast(UnivariateData, Horizon, Aggregation,
CoefficientCombination=NULL, Method = "r", Threshold="hard", Lambda=0.05)
```

Arguments

UnivariateData	[1:n] Numerical vector with n values.
Horizon	Number indicating horizon for forecast from 1 to horizon.
CoefficientCombination	[1:Scales+1] Numerical vector with numbers which are associated with wavelet levels. The last number is associated with the smooth level. Each number determines the number of coefficient used per level. The selection follows a specific scheme.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series.

Method	String indicating which method to use. Available methods: 'r' = Autoregression. 'nn' = Neural Network. 'elm' = Extreme Learning Machine. 'nnetar' = forecast::nnetar. Default: Method="r".
Threshold	Character indicating if Thresholding is done on the wavelet decomposition or not. Default: Threshold="hard". Possible entries: Threshold="hard" for hard thresholding. Threshold="soft" for soft thresholding. Any other input indicates no thresholding.
Lambda	Numeric value indicating the threshold for computing a hard or soft threshold on the wavelet decomposition.

Value

List of

multistep [1:Horizon] Numerical vector with forecast of horizon according to its index.

Author(s)

Quirin Stier

References

[Stier et al., 2021] Stier, Q.,Gehlert, T. and Thrun, M. C.: Multiresolution Forecasting for Industrial Applications, Processess, 2021.

Examples

```
data(AirPassengers)
len_data = length(array(AirPassengers))
UnivariateData = as.vector(AirPassengers)[1:(len_data-1)]
# One-step forecast (Multiresolution Forecast)
one_step = mrf_multi_step_forecast(UnivariateData = UnivariateData,
                                   Horizon = 2,
                                   CoefficientCombination = c(1,1,1),
                                   Aggregation = c(2,4),
                                   Method="r")
# Multi-step forecast (Multiresolution Forecast)
# Horizon = 2 => Forecast with Horizon 1 and 2 as vector
multi_step = mrf_multi_step_forecast(UnivariateData = UnivariateData,
                                     Horizon = 2,
                                     CoefficientCombination = c(1,1,1),
                                     Aggregation = c(2,4),
                                     Method="r")
```

mrf_neuralnet_one_step_forecast

One Step Forecast with Neural Network

Description

This function creates a one step forecast using a multi layer perceptron with one hidden Layer. The number of input is the sum of all coefficients chosen with the parameter CoefficientCombination. The CoefficientCombination parameter controls the number of coefficients chosen for each wavelet and smooth part level individually.

Usage

```
mrf_neuralnet_one_step_forecast(UnivariateData, CoefficientCombination,
Aggregation, Threshold="hard", Lambda=0.05)
```

Arguments

UnivariateData	[1:n] Numerical vector with n values.
CoefficientCombination	[1:Scales+1] Numerical vector with numbers which are associated with wavelet levels. The last number is associated with the smooth level. Each number determines the number of coefficient used per level. The selection follows a specific scheme.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series.
Threshold	Character indicating if Thresholding is done on the wavelet decomposition or not. Default: Threshold="hard". Possible entries: Threshold="hard" for hard thresholding. Threshold="soft" for soft thresholding. Any other input indicates no thresholding.
Lambda	Numeric value indicating the threshold for computing a hard or soft threshold on the wavelet decomposition.

Value

forecast	Numerical value with one step forecast
----------	--

Author(s)

Quirin Stier

References

Aussem, A., Campbell, J., and Murtagh, F. Waveletbased Feature Extraction and Decomposition Strategies for Financial Forecasting. *International Journal of Computational Intelligence in Finance*, 6,5-12, 1998.

Renaud, O., Starck, J.-L., and Murtagh, F. Prediction based on a Multiscale De- composition. *International Journal of Wavelets, Multiresolution and Information Processing*, 1(2):217-232. doi:10.1142/S0219691303000153, 2003.

Murtagh, F., Starck, J.-L., and Renaud, O. On Neuro-Wavelet Modeling. *Decision Support Systems*, 37(4):475-484. doi:10.1016/S0167-9236(03)00092-7, 2004.

Renaud, O., Starck, J.-L., and Murtagh, F. Wavelet-based combined Signal Filter- ing and Prediction. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 35(6):1241-1251. doi:10.1109/TSMCB.2005.850182, 2005.

Examples

```
data(AirPassengers)
len_data = length(as.vector(array(AirPassengers)))
UnivariateData = as.vector(AirPassengers)[1:(len_data-1)]
CoefficientCombination = c(1,1,1)
Aggregation = c(2,4)
if(requireNamespace('monmlp', quietly = TRUE)){
forecast = mrf_neuralnet_one_step_forecast(UnivariateData,
                                           CoefficientCombination,
                                           Aggregation)

true_value = array(AirPassengers)[len_data]
error = true_value - forecast
}
```

mrf_nnetar_forecast *Forecast with nnetar*

Description

This function creates a one step forecast using a multi layer perceptron with one hidden Layer. The number of input is the sum of all coefficients chosen with the parameter `CoefficientCombination`. The `CoefficientCombination` parameter controls the number of coefficients chosen for each wavelet and smooth part level individually.

Usage

```
mrf_nnetar_forecast(UnivariateData, Horizon, Aggregation, Threshold="hard",
                    Lambda=0.05)
```

Arguments

UnivariateData	[1:n] Numerical vector with n values.
Horizon	Number indicating horizon for forecast from 1 to horizon.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series.
Threshold	Character indicating if Thresholding is done on the wavelet decomposition or not. Default: Threshold="hard". Possible entries: Threshold="hard" for hard thresholding. Threshold="soft" for soft thresholding. Any other input indicates no thresholding.
Lambda	Numeric value indicating the threshold for computing a hard or soft threshold on the wavelet decomposition.

Value

forecast	Numerical value with one step forecast
----------	--

Author(s)

Quirin Stier

References

Aussem, A., Campbell, J., and Murtagh, F. Waveletbased Feature Extraction and Decomposition Strategies for Financial Forecasting. *International Journal of Computational Intelligence in Finance*, 6,5-12, 1998.

Renaud, O., Starck, J.-L., and Murtagh, F. Prediction based on a Multiscale De- composition. *International Journal of Wavelets, Multiresolution and Information Processing*, 1(2):217-232. doi:10.1142/S0219691303000153, 2003.

Murtagh, F., Starck, J.-L., and Renaud, O. On Neuro-Wavelet Modeling. *Decision Support Systems*, 37(4):475-484. doi:10.1016/S0167-9236(03)00092-7, 2004.

Renaud, O., Starck, J.-L., and Murtagh, F. Wavelet-based combined Signal Filtering and Prediction. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 35(6):1241-1251. doi:10.1109/TSMCB.2005.850182, 2005.

Examples

```
data(AirPassengers)
len_data = length(as.vector(array(AirPassengers)))
UnivariateData = as.vector(AirPassengers)[1:(len_data-1)]
Aggregation = c(2,4)
if(requireNamespace('nnfor', quietly = TRUE)){
forecast = mrf_nnetar_forecast(UnivariateData, Horizon=1, Aggregation)
true_value = array(AirPassengers)[len_data]
error = true_value - forecast
}
```

mrf_one_step_forecast *mrf_one_step_forecast Step Forecast*

Description

This function creates a one step forecast using the multiresolution forecasting framework based on the manuscript [Stier et al., 2021] which is currently in press.

Usage

```
mrf_one_step_forecast(UnivariateData, Aggregation,
  CoefficientCombination=NULL,
  Method="r", Threshold="hard", Lambda=0.05)
```

Arguments

UnivariateData	[1:n] Numerical vector with n values.
CoefficientCombination	[1:Scales+1] Numerical vector with numbers which are associated with wavelet levels. The last number is associated with the smooth level. Each number determines the number of coefficient used per level. The selection follows a specific scheme.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series.
Method	String indicating which method to use. Available methods: 'r' = Autoregression. 'nn' = Neural Network. 'elm' = Extreme Learning Machine. 'nnetar' = forecast::nnetar. Default: Method="r".
Threshold	Character indicating if Thresholding is done on the wavelet decomposition or not. Default: Threshold="hard". Possible entries: Threshold="hard" for hard thresholding. Threshold="soft" for soft thresholding. Any other input indicates no thresholding.
Lambda	Numeric value indicating the threshold for computing a hard or soft threshold on the wavelet decomposition.

Value

forecast	Numerical value with one step forecast
----------	--

Author(s)

Quirin Stier

References

[Stier et al., 2021] Stier, Q.,Gehlert, T. and Thrun, M. C.: Multiresolution Forecasting for Industrial Applications, Processess, 2021.

Examples

```
data(AirPassengers)
len_data = length(array(AirPassengers))
UnivariateData = as.vector(AirPassengers)[1:(len_data-1)]
forecast = mrf_one_step_forecast(UnivariateData=UnivariateData,
CoefficientCombination=c(1,1,1), Aggregation=c(2,4))
true_value = array(AirPassengers)[len_data]
error = true_value - forecast
```

mrf_regression_lsm_optimization

Least Square Method for Regression

Description

This function computes the weights for the autoregression depending on the given wavelet decomposition. It uses ordinary least square method for optimizing a linear equation system.

Usage

```
mrf_regression_lsm_optimization(points_in_future, lsmatrix)
```

Arguments

points_in_future	n many values of the time series, for which there is an equation from a prediction scheme.
lsmatrix	Matrix carrying predictive equations associated with a specific value of the time series.

Value

List of	
weights	Array of weights carrying the solution for a matrix problem, which was solves with ordinary least squares.

Author(s)

Quirin Stier

References

- Aussem, A., Campbell, J., and Murtagh, F. Waveletbased Feature Extraction and Decomposition Strategies for Financial Forecasting. *International Journal of Computational Intelligence in Finance*, 6,5-12, 1998.
- Renaud, O., Starck, J.-L., and Murtagh, F. Prediction based on a Multiscale De- composition. *International Journal of Wavelets, Multiresolution and Information Processing*, 1(2):217-232. doi:10.1142/S0219691303000153, 2003.
- Murtagh, F., Starck, J.-L., and Renaud, O. On Neuro-Wavelet Modeling. *Decision Support Systems*, 37(4):475-484. doi:10.1016/S0167-9236(03)00092-7, 2004.
- Renaud, O., Starck, J.-L., and Murtagh, F. Wavelet-based combined Signal Filter- ing and Prediction. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 35(6):1241-1251. doi:10.1109/TSMCB.2005.850182, 2005.

Examples

```
data(AirPassengers)
len_data = length(array(AirPassengers))
CoefficientCombination = c(1,1,1)
Aggregation = c(2,4)
UnivariateData = as.vector(AirPassengers)
# Decomposition
dec_res <- wavelet_decomposition(UnivariateData, Aggregation)
# Training
trs_res <- wavelet_training_equations(UnivariateData,
                                     dec_res$WaveletCoefficients,
                                     dec_res$SmoothCoefficients,
                                     dec_res$Scales,
                                     CoefficientCombination, Aggregation)

arr_future_points = trs_res$points_in_future
matrix = trs_res$lsmatrix
# Optimization method
weights = mrf_regression_lsm_optimization(arr_future_points, matrix)
# Forecast
scheme = wavelet_prediction_equation(dec_res$WaveletCoefficients,
                                     dec_res$SmoothCoefficients, CoefficientCombination, Aggregation)
forecast = weights
```

mrf_regression_one_step_forecast

One Step Forecast with Regression

Description

This function creates a one step forecast using an autoregression method. The `ccps` parameter controls the number of coefficients chosen for each wavelet and smooth part level individually.

Usage

```
mrf_regression_one_step_forecast(UnivariateData, CoefficientCombination,
Aggregation, Threshold="hard", Lambda=0.05)
```

Arguments

UnivariateData	[1:n] Numerical vector with n values.
CoefficientCombination	[1:Scales+1] Numerical vector with numbers which are associated with wavelet levels. The last number is associated with the smooth level. Each number determines the number of coefficient used per level. The selection follows a specific scheme.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series.
Threshold	Character indicating if Thresholding is done on the wavelet decomposition or not. Default: Threshold="hard". Possible entries: Threshold="hard" for hard thresholding. Threshold="soft" for soft thresholding. Any other input indicates no thresholding.
Lambda	Numeric value indicating the threshold for computing a hard or soft threshold on the wavelet decomposition.

Value

forecast	Numerical value with one step forecast
----------	--

Author(s)

Quirin Stier

References

- Aussem, A., Campbell, J., and Murtagh, F. Waveletbased Feature Extraction and Decomposition Strategies for Financial Forecasting. *International Journal of Computational Intelligence in Finance*, 6,5-12, 1998.
- Renaud, O., Starck, J.-L., and Murtagh, F. Prediction based on a Multiscale De- composition. *International Journal of Wavelets, Multiresolution and Information Processing*, 1(2):217-232. doi:10.1142/S0219691303000153, 2003.
- Murtagh, F., Starck, J.-L., and Renaud, O. On Neuro-Wavelet Modeling. *Decision Support Systems*, 37(4):475-484. doi:10.1016/S0167-9236(03)00092-7, 2004.
- Renaud, O., Starck, J.-L., and Murtagh, F. Wavelet-based combined Signal Filtering and Prediction. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 35(6):1241-1251. doi:10.1109/TSMCB.2005.850182, 2005.

Examples

```

data(AirPassengers)
len_data = length(as.vector(array(AirPassengers)))
UnivariateData = as.vector(AirPassengers)[1:(len_data-1)]
CoefficientCombination = c(1,1,1)
Aggregation = c(2,4)
forecast = mrf_regression_one_step_forecast(UnivariateData,
                                           CoefficientCombination,
                                           Aggregation)

true_value = array(AirPassengers)[len_data]
error = true_value - forecast

```

mrf_requirement	<i>Multiresolution Forecast Requirements</i>
-----------------	--

Description

Computes requirements for given model using insights of various published papers and the manuscript [Stier et al., 2021] which is currently in press.

Usage

```
mrf_requirement(UnivariateData, CoefficientCombination, Aggregation)
```

Arguments

UnivariateData	[1:n] Numerical vector with n values.
CoefficientCombination	[1:Scales+1] Numerical vector with numbers which are associated with wavelet levels. The last number is associated with the smooth level. Each number determines the number of coefficient used per level. The selection follows a specific scheme.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series.

Value

List of	
MinLen	Integer minimum required length for model.
StartTraining	Integer indicating the index of time series at which the training equations can be built up.
NumberWeights	Number of weights required for building model.
NumberEquations	Number of equations which can be built with given data.

Author(s)

Quirin Stier

References

[Stier et al., 2021] Stier, Q., Gehlert, T. and Thrun, M. C.: Multiresolution Forecasting for Industrial Applications, *Processess*, 2021.

Aussem, A., Campbell, J., and Murtagh, F. Waveletbased Feature Extraction and Decomposition Strategies for Financial Forecasting. *International Journal of Computational Intelligence in Finance*, 6,5-12, 1998.

Renaud, O., Starck, J.-L., and Murtagh, F. Prediction based on a Multiscale De- composition. *International Journal of Wavelets, Multiresolution and Information Processing*, 1(2):217-232. doi:10.1142/S0219691303000153, 2003.

Murtagh, F., Starck, J.-L., and Renaud, O. On Neuro-Wavelet Modeling. *Decision Support Systems*, 37(4):475-484. doi:10.1016/S0167-9236(03)00092-7, 2004.

Renaud, O., Starck, J.-L., and Murtagh, F. Wavelet-based combined Signal Filtering and Prediction. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 35(6):1241-1251. doi:10.1109/TSMCB.2005.850182, 2005.

Examples

```
data(entsoe)
UnivariateData = entsoe$value
mrf_requirement(UnivariateData, c(2,3,4), c(2,4))
```

mrf_rolling_forecasting_origin

Rolling forecasting origin for Multiresolution Forecasts

Description

This function computes a rolling forecasting origin for one- or multi-step forecasts with a specific method based on the manuscript [Stier et al., 2021] which is currently in press. Multi-step forecasts are computed recursively with the one step forecast method.

Usage

```
mrf_rolling_forecasting_origin(UnivariateData, Aggregation,
CoefficientCombination=NULL, Horizon = 2, Window = 3, Method = "r",
NumClusters = 1,
Threshold="hard", Lambda=0.05)
```

Arguments

UnivariateData	[1:n] Numerical vector with n values.
CoefficientCombination	[1:Scales+1] Numerical vector with numbers which are associated with wavelet levels. The last number is associated with the smooth level. Each number determines the number of coefficient used per level. The selection follows a specific scheme.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series.
Horizon	Number indicating horizon for forecast from 1 to horizon.
Window	Number indicating how many points are used for cross validation.
Method	String indicating which method to use. Available methods: 'r' = Autoregression. 'nn' = Neural Network. 'elm' = Extreme Learning Machine. 'nnetar' = forecast::nnetar. Default: Method="r".
NumClusters	Number of clusters used for parallel computing.
Threshold	Character indicating if Thresholding is done on the wavelet decomposition or not. Default: Threshold="hard". Possible entries: Threshold="hard" for hard thresholding. Threshold="soft" for soft thresholding. Any other input indicates no thresholding.
Lambda	Numeric value indicating the threshold for computing a hard or soft threshold on the wavelet decomposition.

Details

Thus, h-step forecast for $h = 1, \dots$, horizon for window_size many steps can be computed. The forecasting method can be an autoregression or a neural network (multilayer perceptron). The CoefficientCombination parameter controls the number of coefficients chosen for each wavelet and smooth part level individually. The NumClusters parameter determines the number of cluster used for parallel computation. NumClusters = 1 performs a non parallel version. NumClusters is constrained by the maximum number of clusters available minus one to prevent the machine to be overchallenged.

Value

List of	
Error	[1:Window,1:Horizon] Numerical Matrix with 'Window' many row entries indicating one time point with 'Horizon' many forecast errors.
Forecast	[1:Window,1:Horizon] Numerical Matrix with 'Window' many row entries indicating one time point with 'Horizon' many forecasts.

Author(s)

Quirin Stier

References

[Stier et al., 2021] Stier, Q., Gehlert, T. and Thrun, M. C.: Multiresolution Forecasting for Industrial Applications, *Processess*, 2021.

Examples

```
data(AirPassengers)
UnivariateData=as.vector(array(AirPassengers))
res = mrf_rolling_forecasting_origin(UnivariateData,
                                   CoefficientCombination = c(10,10,10),
                                   Aggregation = c(2,4),
                                   Horizon = 2, Window = 3, Method = "r",
                                   NumClusters = 1)

Error = res$Error
Forecast = res$Forecast
```

mrf_train

Multiresolution Forecast

Description

Creates a multiresolution forecast model which can be used for forecasting with method 'mrf_forecast' based on the manuscript [Stier et al., 2021] which is currently in press.

Usage

```
mrf_train(Data, Horizon=1, Aggregation="auto", Method = "r",
           TimeSteps4ModelSelection=2, crit="AIC", InSample=FALSE, Threshold="hard",
           Lambda=0.05, NumClusters=1, itermax=1)
```

Arguments

Data	[1:n] Numerical vector with n values from the training data.
Horizon	Number indicating forecast horizon. Horizon = 1 means one-step forecast and Horizon > 1 means a one-step forecast and all multi-step forecasts from horizon 2 to 'Horizon'. Default: Horizon = 1.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series. Default: Aggregation = "auto".
Method	String indicating which method to use. Available methods: 'r' = Autoregression. 'nn' = Neural Network. 'elm' = Extreme Learning Machine. 'nnetar' = forecast::nnetar. Default: Method="r".
TimeSteps4ModelSelection	Number of time steps of data (newest part) on which a model selection is performed. Default: TimeSteps4ModelSelection = 2.
crit	String with criterion. Available criterions: "AIC" = Akaike's Info. Crit. "MAE" = Mean Abs. Error. "MRE" = Mean Root Error. Default: crit = "AIC".

InSample	Boolean, deciding if in-sample-forecast based on rolling forecasting origin is computed or not. TRUE = Computation of in-sample-forecast. FALSE = No computation. Default: InSample = FALSE
Threshold	Character indicating if Thresholding is done on the wavelet decomposition or not. Default: Threshold="hard". Possible entries: Threshold="hard" for hard thresholding. Threshold="soft" for soft thresholding. Any other input indicates no thresholding.
Lambda	Numeric value indicating the threshold for computing a hard or soft threshold on the wavelet decomposition.
NumClusters	Number of clusters used for parallel computing. Default: NumClusters = 1.
itermax	Number of iterations used in the differential evolutionary optimization algorithm. Default: itermax = 1.

Value

List with	
Data	[1:n] Numerical vector with n values from the training data.
Method	String indicating which method to use.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series.
CoefficientCombination	[1:Scales+1] Numerical vector with numbers which are associated with wavelet levels. The last number is associated with the smooth level. Each number determines the number of coefficient used per level. The selection follows a specific scheme.
Horizon	Number indicating forecast horizon. Horizon = 1 means one-step forecast and Horizon > 1 means a one-step forecast and all multi-step forecasts from horizon 2 to 'Horizon'.
ModelError	[1:TimeSteps4ModelSelection, 1:Horizon] Numerical matrix with one-/multi-steps in columns and the time steps rowwise. The error is according to the scheme of a rolling forecasting origin. The length depends on the minimum required length for constructing the wavelet model and the length of data. The newer part of the data is used for the model fit truncating the oldest data according to the minimum required length for constructing the model.
ModelMAE	Integer: Mean Absolute Error (MAE) computed for the in-sample-forecast resulting from a rolling forecasting origin.

Author(s)

Quirin Stier

References

[Stier et al., 2021] Stier, Q.,Gehlert, T. and Thrun, M. C.: Multiresolution Forecasting for Industrial Applications, Processess, 2021.

Examples

```

data(AirPassengers)
Data = as.vector(AirPassengers)
len_data = length(Data)
Train = Data[1:(len_data-2)]
Test = Data[(len_data-1):len_data]
# One-step forecast (Multiresolution Forecast)
model = mrf_train(Train)
one_step = mrf_forecast(model, Horizon=1)
Error = one_step$Forecast - Test[1]
# Multi-step forecast (Multiresolution Forecast)
# Horizon = 2 => Forecast with Horizon 1 and 2 as vector
model = mrf_train(Train, Horizon=2)
multi_step = mrf_forecast(model, Horizon=2)
Error = multi_step$Forecast - Test

```

wavelet_decomposition *Redundant Haar Wavelet Decomposition*

Description

This function decomposes a time series in its wavelet and smooth coefficients using the redundant Haar wavelet transform.

Usage

```

wavelet_decomposition(UnivariateData, Aggregation = c(2, 4, 8, 16, 32),
Threshold="hard", Lambda=0.05)

```

Arguments

UnivariateData	[1:n] Numerical vector with n time series values
Aggregation	[1:Scales] Numerical vector of length 'Scales' carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of values used for aggregation from the original time series.
Threshold	Character indicating if Thresholding is done on the wavelet decomposition or not. Default: Threshold="hard". Possible entries: Threshold="hard" for hard thresholding. Threshold="soft" for soft thresholding. Any other input indicates no thresholding.
Lambda	Numeric value indicating the threshold for computing a hard or soft threshold on the wavelet decomposition.

Details

The resulting wavelet and smooth coefficients are stored in so called wavelet and smooth part levels. The smooth part level is created from the original times series by aggregation (average). This makes the times series in some sense smoother, hence the naming. Each individual smooth part level can be created from the original time series by aggregating over different number of values. The different smooth part levels are ordered, so that the number of values used for aggregation are ascending. A dyadic scheme is recommended (increasing sequences of the power of two). The dyadic scheme for 5 levels would require $\text{agg_per_lvl} = c(2, 4, 8, 16, 32)$. So the first smooth part level would be the average of two points of the original time series, the second smooth part level would be the average of four points, and so on. This averaging is applied asymmetrical. That means, that the result of the average of a sequence of points is obtained for the last point in time of that sequence. So each smooth part level starts with a certain offset, since no average can be obtained for the first particular points in time. The wavelet levels are the differences between the original time series and the smooth levels. The first wavelet level is the difference of the original time series and the first smooth part level. The second wavelet level is the difference of the first and second smooth part level and so on.

Value

List of

UnivariateData [1:n] Numerical vector with n time series values.

WaveletCoefficients

[Scales, n] Matrix with 'Scales' many wavelet scales row-wise with n columns corresponding to the time domain of a time series.

SmoothCoefficients

[Scales, n] Matrix with 'Scales' many smooth approximation scales row-wise with n columns corresponding to the time domain of a time series.

Scales Number of wavelet levels.

Author(s)

Quirin Stier

References

Aussem, A., Campbell, J., and Murtagh, F. Waveletbased Feature Extraction and Decomposition Strategies for Financial Forecasting. *International Journal of Computational Intelligence in Finance*, 6,5-12, 1998.

Renaud, O., Starck, J.-L., and Murtagh, F. Prediction based on a Multiscale De- composition. *International Journal of Wavelets, Multiresolution and Information Processing*, 1(2):217-232. doi:10.1142/S0219691303000153, 2003.

Murtagh, F., Starck, J.-L., and Renaud, O. On Neuro-Wavelet Modeling. *Decision Support Systems*, 37(4):475-484. doi:10.1016/S0167-9236(03)00092-7, 2004.

Renaud, O., Starck, J.-L., and Murtagh, F. Wavelet-based combined Signal Filtering and Prediction. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 35(6):1241-1251. doi:10.1109/TSMCB.2005.850182, 2005.

Examples

```

data(AirPassengers)
plot(AirPassengers, type = "l", col = "black")
UnivariateData = as.vector(array(AirPassengers))
dec_res = wavelet_decomposition(UnivariateData, Aggregation = c(2,4))
plot(dec_res$SmoothCoefficients[2,4:length(dec_res$SmoothCoefficients[2,])],
type = "l", col = "blue")
lines(array(AirPassengers)[4:length(dec_res$SmoothCoefficients[2,])],
col = "black")

```

wavelet_prediction_equation

One Step Forecast with Regression

Description

This function delivers the required wavelet and smooth coefficients from the decomposition based on a prediction scheme.

Usage

```

wavelet_prediction_equation(WaveletCoefficients, SmoothCoefficients,
CoefficientCombination, Aggregation)

```

Arguments

WaveletCoefficients	[Scales, n] Matrix with 'Scales' many wavelet scales row-wise with n columns corresponding to the time domain of a time series.
SmoothCoefficients	[Scales, n] Matrix with 'Scales' many smooth approximation scales row-wise with n columns corresponding to the time domain of a time series.
CoefficientCombination	[1:Scales+1] Numerical vector with numbers which are associated with wavelet levels. The last number is associated with the smooth level. Each number determines the number of coefficient used per level. The selection follows a specific scheme.
Aggregation	[1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series.

Value

future_point	Numerical value carrying one step forecast.
--------------	---

Author(s)

Quirin Stier

References

- Aussem, A., Campbell, J., and Murtagh, F. Waveletbased Feature Extraction and Decomposition Strategies for Financial Forecasting. *International Journal of Computational Intelligence in Finance*, 6,5-12, 1998.
- Renaud, O., Starck, J.-L., and Murtagh, F. Prediction based on a Multiscale De- composition. *International Journal of Wavelets, Multiresolution and Information Processing*, 1(2):217-232. doi:10.1142/S0219691303000153, 2003.
- Murtagh, F., Starck, J.-L., and Renaud, O. On Neuro-Wavelet Modeling. *Decision Support Systems*, 37(4):475-484. doi:10.1016/S0167-9236(03)00092-7, 2004.
- Renaud, O., Starck, J.-L., and Murtagh, F. Wavelet-based combined Signal Filter- ing and Prediction. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 35(6):1241-1251. doi:10.1109/TSMCB.2005.850182, 2005.

Examples

```
data(AirPassengers)
len_data = length(array(AirPassengers))
CoefficientCombination = c(1,1,1)
Aggregation = c(2,4)
UnivariateData = as.vector(AirPassengers)
# Decomposition
dec_res <- wavelet_decomposition(UnivariateData, Aggregation)
# Training
trs_res <- wavelet_training_equations(UnivariateData,
                                     dec_res$WaveletCoefficients,
                                     dec_res$SmoothCoefficients,
                                     dec_res$Scales,
                                     CoefficientCombination, Aggregation)

arr_future_points = trs_res$points_in_future
matrix = trs_res$lsmatrix
# Optimization method
weights = mrf_regression_lsm_optimization(arr_future_points, matrix)
# Forecast
scheme = wavelet_prediction_equation(dec_res$WaveletCoefficients,
                                     dec_res$SmoothCoefficients, CoefficientCombination, Aggregation)
forecast = weights
```

wavelet_training_equations

Generic Training Scheme for wavelet framework

Description

This function computes the input for the training phase required for one step forecasts. This computational step is required for all one step forecast procedures contained in this package.

Usage

```
wavelet_training_equations(UnivariateData, WaveletCoefficients,
SmoothCoefficients, Scales, CoefficientCombination, Aggregation)
```

Arguments

UnivariateData [1:n] Numerical vector with n values.

WaveletCoefficients [Scales, n] Matrix with 'Scales' many wavelet scales row-wise with n columns corresponding to the time domain of a time series.

SmoothCoefficients [Scales, n] Matrix with 'Scales' many smooth approximation scales row-wise with n columns corresponding to the time domain of a time series.

Scales Number of wavelet levels.

CoefficientCombination [1:Scales+1] Numerical vector with numbers which are associated with wavelet levels. The last number is associated with the smooth level. Each number determines the number of coefficient used per level. The selection follows a specific scheme.

Aggregation [1:Scales] Numerical vector carrying numbers whose index is associated with the wavelet level. The numbers indicate the number of time in points used for aggregation from the original time series.

Value

points_in_future n many values of the time series, for which there is an equation from a prediction scheme.

lsmatrix Matrix carrying predictive equations associated with a specific value of the time series.

Author(s)

Quirin Stier

References

- Aussem, A., Campbell, J., and Murtagh, F. Waveletbased Feature Extraction and Decomposition Strategies for Financial Forecasting. *International Journal of Computational Intelligence in Finance*, 6,5-12, 1998.
- Renaud, O., Starck, J.-L., and Murtagh, F. Prediction based on a Multiscale De- composition. *International Journal of Wavelets, Multiresolution and Information Processing*, 1(2):217-232. doi:10.1142/S0219691303000153, 2003.
- Murtagh, F., Starck, J.-L., and Renaud, O. On Neuro-Wavelet Modeling. *Decision Support Systems*, 37(4):475-484. doi:10.1016/S0167-9236(03)00092-7, 2004.
- Renaud, O., Starck, J.-L., and Murtagh, F. Wavelet-based combined Signal Filtering and Prediction. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 35(6):1241-1251. doi:10.1109/TSMCB.2005.850182, 2005.

Examples

```
data(AirPassengers)
len_data = length(array(AirPassengers))
CoefficientCombination = c(1,1,1)
Aggregation = c(2,4)
UnivariateData = as.vector(AirPassengers)
# Decomposition
dec_res <- wavelet_decomposition(UnivariateData, Aggregation)
# Training
trs_res <- wavelet_training_equations(UnivariateData,
                                     dec_res$WaveletCoefficients,
                                     dec_res$SmoothCoefficients,
                                     dec_res$Scales,
                                     CoefficientCombination, Aggregation)

arr_future_points = trs_res$points_in_future
matrix = trs_res$lsmatrix
# Optimization method
weights = mrf_regression_lsm_optimization(arr_future_points, matrix)
# Forecast
scheme = wavelet_prediction_equation(dec_res$WaveletCoefficients,
                                     dec_res$SmoothCoefficients, CoefficientCombination, Aggregation)
forecast = weights
```

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