

# Package ‘monreg’

April 26, 2020

**Version** 0.1.4

**Date** 2020-04-25

**Title** Nonparametric Monotone Regression

**Author** This version Kay Pilz and Steffanie Titoff. Earlier developements by Holger Dette and Kay Pilz.

**Maintainer** Scott Kostyshak <scott.kostyshak@gmail.com>

**Depends** R (>= 2.0.0)

**Description** Estimates monotone regression and variance functions in a nonparametric model, based on Dette, Holger, Neumeyer, and Pilz (2006) <doi:10.3150/bj/1151525131>.

**Encoding** latin1

**License** GPL (>= 2)

**NeedsCompilation** yes

**URL** <https://gitlab.com/scottkosty/monreg>

**BugReports** <https://gitlab.com/scottkosty/monreg/issues>

**Repository** CRAN

**Date/Publication** 2020-04-26 07:00:09 UTC

## R topics documented:

|                       |   |
|-----------------------|---|
| monreg . . . . .      | 2 |
| monvardiff . . . . .  | 4 |
| monvarresid . . . . . | 6 |

|              |          |
|--------------|----------|
| <b>Index</b> | <b>9</b> |
|--------------|----------|

monreg

*Estimating Monotone Regression Functions Nonparametrically***Description**

monreg provides a strictly monotone estimator of the regression function based on the nonparametric regression model.

**Usage**

```
monreg(x, y, a = min(x), b = max(x), N = length(x), t = length(x),
      hd, Kd = "epanech", hr, Kr = "epanech", degree = 1,
      inverse = 0, monotonie = "isoton")
```

**Arguments**

|           |                                                                                                                                                                                                                                                                             |
|-----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x         | vector containing the x-values (design points) of a sample                                                                                                                                                                                                                  |
| y         | vector containing the y-values (response) of a sample                                                                                                                                                                                                                       |
| a         | lower bound of the support of the design points density function, or smallest fixed design point                                                                                                                                                                            |
| b         | upper bound of the support of the design points density function, or largest fixed design point                                                                                                                                                                             |
| N         | number or vector of evaluation points of the unconstrained nonparametric regression estimator (e.g. Nadaraya-Watson estimator)                                                                                                                                              |
| t         | number or vector of points where the monotone estimation is computed                                                                                                                                                                                                        |
| hd        | bandwith of kernel $K_d$ of the density estimation step                                                                                                                                                                                                                     |
| Kd        | Kernel for the density estimation step (monotonization step). 'epanech' for Epanechnikov, 'rectangle' for rectangle, 'biweight' for biweight, 'triweight' for triweight, 'triangle' for triangle, 'cosine' for cosine kernel                                                |
| hr        | bandwith of kernel $K_r$ of the regression estimation step.                                                                                                                                                                                                                 |
| Kr        | Kernel for the regression estimation step (unconstrained estimation). 'epanech' for Epanechnikov, 'rectangle' for rectangle, 'biweight' for biweight, 'triweight' for triweight, 'triangle' for triangle, 'cosine' for cosine kernel.                                       |
| degree    | Determines the method for the unconstrained estimation. '0' for the classical Nadaraya-Watson estimate, '1' for the local linear estimate. As well degree can be the vector of the unconditional estimator provided by the user for the design points given in the vector N |
| inverse   | For '0' the original regression function is estimated, for '1' the inverse of the regression function is estimated.                                                                                                                                                         |
| monotonie | Determines the type of monotonicity. 'isoton' if the regression function is assumed to be isotone, 'antinton' if the regression function is assumed to be anti-tone.                                                                                                        |

## Details

Nonparametric regression models are of the form  $Y_i = m(X_i) + \sigma(X_i) \cdot \varepsilon_i$ , where  $m$  is the regression function and  $\sigma$  the variance function. `monreg` performs a monotone estimate of the unknown regression function  $m$ . `monreg` first estimates  $m$  by an unconstrained nonparametric method, the classical Nadaraya-Watson estimate or the local-linear estimate (unless the user decides to pass his or her own estimate). In a second step the inverse of the (monotone) regression function is calculated, by monotoneizing this unconstrained estimate. With the above notation and  $\hat{m}$  for the unconstrained estimate, the second step writes as follows,

$$\hat{m}_I^{-1} = \frac{1}{Nh_d} \sum_{i=1}^N \int_{-\infty}^t K_d \left( \frac{\hat{m}(\frac{i}{N}) - u}{h_d} \right) du.$$

Finally, the monotone estimate achieved by inversion of  $\hat{m}_I^{-1}$ .

## Value

`monreg` returns a list of values

|                          |                                                                                                                                                                                                               |
|--------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>xs</code>          | the input values <code>x</code> , standardized on the interval $[0, 1]$                                                                                                                                       |
| <code>y</code>           | input variable <code>y</code>                                                                                                                                                                                 |
| <code>z</code>           | the points, for which the unconstrained function is estimated                                                                                                                                                 |
| <code>t</code>           | the points, for which the monotone function values will be estimated                                                                                                                                          |
| <code>length.x</code>    | length of the vector <code>x</code>                                                                                                                                                                           |
| <code>length.z</code>    | length of the vector <code>z</code>                                                                                                                                                                           |
| <code>length.t</code>    | length of the vector <code>t</code>                                                                                                                                                                           |
| <code>hd</code>          | bandwidth used with the Kernel $K_d$                                                                                                                                                                          |
| <code>hr</code>          | bandwidth used with the Kernel $K_r$                                                                                                                                                                          |
| <code>Kd</code>          | kernel used for the monotone step                                                                                                                                                                             |
| <code>Kr</code>          | kernel used for the initial unconstrained regression estimate                                                                                                                                                 |
| <code>degree</code>      | method, which was used for the unconstrained regression estimate                                                                                                                                              |
| <code>ldeg.vektor</code> | length of the vector <code>degree</code> . If <code>ldeg.vektor</code> is not equal to 1 the user provided the vector of the unconditional estimator for the design points given in the vector <code>N</code> |
| <code>inverse</code>     | indicates, if the origin regression function or its inverse has been estimated                                                                                                                                |
| <code>estimation</code>  | the monotone estimate at the design points <code>t</code>                                                                                                                                                     |
| <code>...</code>         |                                                                                                                                                                                                               |

## Author(s)

This R Package was developed by Kay Pilz and Stefanie Titoff. Earlier developements of the estimator were made by Holger Dette and Kay Pilz.

## See Also

`monvardiff` and `monvarresid` for monotone variance function estimation.

## Examples

```
x <- rnorm(100)
y <- x + rnorm(100)
mon1 <- monreg(x, y, hd = .5, hr = .5)
plot(mon1$t, mon1$estimation)
```

---

monvardiff

*Estimating Monotone Variance Functions Using Pseudo-Residuals*

---

## Description

monvardiff provides a strictly monotone estimator of the variance function based on the nonparametric regression model.

## Usage

```
monvardiff(x, y, a=min(x), b=max(x), N=length(x), t=length(x), r=2, hr, Kr="epanech",
           hd, Kd="epanech", degree=1, inverse=0, monotonic="isoton")
```

## Arguments

|        |                                                                                                                                                                                                                                                                                      |
|--------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x      | vector containing the x-values (design points) of a sample                                                                                                                                                                                                                           |
| y      | vector containing the y-values (response) of a sample                                                                                                                                                                                                                                |
| a      | lower bound of the support of the design points density function, or smallest fixed design point                                                                                                                                                                                     |
| b      | upper bound of the support of the design points density function, or largest fixed design point                                                                                                                                                                                      |
| N      | number or vector of evaluation points of the unconstrained nonparametric variance estimator (e.g. Nadaraya-Watson estimator)                                                                                                                                                         |
| t      | number or vector of points where the monotone estimation is computed                                                                                                                                                                                                                 |
| r      | order of the difference scheme, i.e. weights $d_0, \dots, d_r$ to calculate the pseudo-residuals                                                                                                                                                                                     |
| hr     | bandwidth of kernel $K_r$ of the variance estimation step                                                                                                                                                                                                                            |
| Kr     | Kernel for the variance estimation step (unconstrained estimation). 'epanech' for Epanechnikov, 'rectangle' for rectangle, 'biweight' for biweight, 'triweight' for triweight, 'triangle' for triangle, 'cosine' for cosine kernel                                                   |
| hd     | bandwidth of kernel $K_d$ of the density estimation step                                                                                                                                                                                                                             |
| Kd     | Kernel for the density estimation step (monotonization step). 'epanech' for Epanechnikov, 'rectangle' for rectangle, 'biweight' for biweight, 'triweight' for triweight, 'triangle' for triangle, 'cosine' for cosine kernel                                                         |
| degree | determines the method for the unconstrained variance estimation. '0' for the classical Nadaraya-Watson estimate, '1' for the local linear estimate. As well degree can be the vector of the unconditional estimator provided by the user for the design points given in the vector N |

|           |                                                                                                                                                                 |
|-----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| inverse   | for '0' the original variance function is estimated, for '1' the inverse of the variance function is estimated.                                                 |
| monotonie | determines the type of monotonicity. 'isoton' if the variance function is assumed to be isotone, 'antinton' if the variance function is assumed to be antitone. |

### Details

Nonparametric regression models are of the form  $Y_i = m(X_i) + \sigma(X_i) \cdot \varepsilon_i$ , where  $m$  is the regression function and  $\sigma$  the variance function. `monvardiff` performs a monotone estimate of the unknown variance function  $s = \sigma^2$ . `monvardiff` first estimates  $s$  by an unconstrained nonparametric method, the classical Nadaraya-Watson estimate or the local-linear estimate (unless the user decides to pass his or her own estimate). This estimation contains the usage of the Pseudo-Residuals. In a second step the inverse of the (monotone) variance function is calculated by monotoneizing the unconstrained estimate from the first step. With the above notation and  $\hat{s}$  for the unconstrained estimate, the second step writes as follows,

$$\hat{s}_I^{-1} = \frac{1}{Nh_d} \sum_{i=1}^N \int_{-\infty}^t K_d\left(\frac{\hat{s}(\frac{i}{N}) - u}{h_d}\right) du.$$

Finally, the monotone estimate is achieved by inversion of  $\hat{s}_I^{-1}$ .

### Value

`monvardiff` returns a list of values

|             |                                                                                                                                                                                             |
|-------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| xs          | the input values x, standardized on the interval [0, 1]                                                                                                                                     |
| y           | input variable y                                                                                                                                                                            |
| z           | the points, for which the unconstrained function is estimated                                                                                                                               |
| t           | the points, for which the monotone variance function will be estimated                                                                                                                      |
| length.x    | length of the vector x                                                                                                                                                                      |
| length.z    | length of the vector z                                                                                                                                                                      |
| length.t    | length of the vector t                                                                                                                                                                      |
| r           | order of the difference scheme, i.e. number of weights to calculate the pseudo-residuals                                                                                                    |
| hr          | bandwidth used with the Kernel $K_r$                                                                                                                                                        |
| hd          | bandwidth used with the Kernel $K_d$                                                                                                                                                        |
| Kr          | kernel used for the unconstrained variance estimate                                                                                                                                         |
| Kd          | kernel used for the monotone step                                                                                                                                                           |
| degree      | method, which was used for the unconstrained variance estimate                                                                                                                              |
| ldeg.vektor | length of the vector degree. If <code>ldeg.vektor</code> is not equal to 1 the user provided the vector of the unconditional variance estimator for the design points given in the vector N |
| inverse     | indicates, if the origin variance function or its inverse has been estimated                                                                                                                |
| estimation  | the monotone estimate at the design points $t$                                                                                                                                              |

**Author(s)**

This R Package was developed by Kay Pilz and Stefanie Titoff. Earlier developments of the estimator were made by Holger Dette and Kay Pilz.

**See Also**

monreg for monotone regression function estimation and monvarresid for monotone variance function estimation by nonparametric residuals.

---

|             |                                                                             |
|-------------|-----------------------------------------------------------------------------|
| monvarresid | <i>Estimating Monotone Variance Functions Using Nonparametric Residuals</i> |
|-------------|-----------------------------------------------------------------------------|

---

**Description**

monvarresid provides a strictly monotone estimator of the variance function based on the nonparametric regression model.

**Usage**

```
monvarresid(x,y,a=min(x),b=max(x),N=length(x),t=length(x),h,K="epanech",hd,Kd="epanech",
hr,Kr="epanech",mdegree=1,sdegree=1,inverse=0,monotonie="isoton")
```

**Arguments**

|    |                                                                                                                                                                                                                               |
|----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x  | vector containing the x-values (design points) of a sample                                                                                                                                                                    |
| y  | vector containing the y-values (response) of a sample                                                                                                                                                                         |
| a  | lower bound of the support of the design points density function, or smallest fixed design point                                                                                                                              |
| b  | upper bound of the support of the design points density function, or largest fixed design point                                                                                                                               |
| N  | number or vector of evaluation points of the unconstrained nonparametric variance estimator (e.g. Nadaraya-Watson estimator)                                                                                                  |
| t  | number or vector of points where the monotone estimation is computed                                                                                                                                                          |
| h  | bandwidth of kernel $K$ of the regression estimation step                                                                                                                                                                     |
| K  | Kernel for the regression estimation step. 'epanech' for Epanechnikov, 'rectangle' for rectangle, 'biweight' for biweight, 'triweight' for triweight, 'triangle' for triangle, 'cosine' for cosine kernel                     |
| hd | bandwidth of kernel $K_d$ of the density estimation step                                                                                                                                                                      |
| Kd | Kernel for the density estimation step (monotonization step). 'epanech' for "Epanechnikov, 'rectangle' for rectangle, 'biweight' for biweight, 'triweight' for triweight, 'triangle' for triangle, 'cosine' for cosine kernel |
| hr | bandwidth of kernel $K_r$ of the variance estimation step                                                                                                                                                                     |

|           |                                                                                                                                                                                                                                                                                       |
|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Kr        | Kernel for the variance estimation step (unconstrained estimation). 'epanech' for "Epanechnikov, 'rectangle' for rectangle, 'biweight' for biweight, 'triweight' for triweight, 'triangle' for triangle, 'cosine' for cosine kernel.                                                  |
| mdegree   | determines the method for the regression estimation. '0' for the classical Nadaraya-Watson estimate, '1' for the local linear estimate. As well mdegree can be the vector of the estimator provided by the user for the design points given by the vector x                           |
| sdegree   | Determines the method for the unconstrained variance estimation. '0' for the classical Nadaraya-Watson estimate, '1' for the local linear estimate. As well sdegree can be the vector of the unconditional estimator provided by the user for the design points given by the vector N |
| inverse   | For '0' the original variance function is estimated, for '1' the inverse of the variance function is estimated.                                                                                                                                                                       |
| monotonie | Determines the type of monotonicity. 'isoton' if the variance function is assumed to be isotone, 'antinton' if the variance function is assumed to be antitone.                                                                                                                       |

### Details

Nonparametric regression models are of the form  $Y_i = m(X_i) + \sigma(X_i) \cdot \varepsilon_i$ , where  $m$  is the regression function and  $\sigma$  the variance function. `monvarresid` performs a monotone estimate of the unknown variance function  $s = \sigma^2$ . `monvarresid` first estimates  $m$  by an unconstrained nonparametric method, the classical Nadaraya-Watson estimate or the local-linear estimate (unless the user decides to pass his or her own estimate). In a second step an unconstrained estimation for  $s$  is performed, again by the classical Nadaraya-Watson method or the local-linear estimate (unless the user decides to pass his or her own estimate). In a third step the inverse of the (monotone) variance function is calculated, by monotoneizing the unconstrained estimate from the second step. With the above notation and  $\hat{s}$  for the unconstrained estimate, the third step writes as follows,

$$\hat{s}_I^{-1} = \frac{1}{N h_d} \sum_{i=1}^N \int_{-\infty}^t K_d \left( \frac{\hat{s}(\frac{i}{N}) - u}{h_d} \right) du.$$

Finally, the monotone estimate is achieved by inversion of  $\hat{s}_I^{-1}$ .

### Value

`monvarresid` returns a list of values

|          |                                                                        |
|----------|------------------------------------------------------------------------|
| xs       | the input values x, standardized on the interval [0, 1]                |
| y        | input variable y                                                       |
| z        | the points, for which the unconstrained function is estimated          |
| t        | the points, for which the monotone variance function will be estimated |
| length.x | length of the vector x                                                 |
| length.z | length of the vector z                                                 |
| length.t | length of the vector t                                                 |

|            |                                                                                                                                                                             |
|------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| h          | bandwidth used with the Kernel $K$                                                                                                                                          |
| hd         | bandwidth used with the Kernel $K_d$                                                                                                                                        |
| hr         | bandwidth used with the Kernel $K_r$                                                                                                                                        |
| K          | kernel used for the regression estimation step                                                                                                                              |
| Kd         | kernel used for the monotone estimation step                                                                                                                                |
| Kr         | kernel used for the unconstrained variance estimate                                                                                                                         |
| mdegree    | method, which was used for the unconstrained regression estimate                                                                                                            |
| lmdeg      | length of the vector mdegree. If lmdeg is not equal to 1 the user provided the vector of the unconditional regression estimator for the design points given by the vector x |
| sdegree    | method, which was used for the unconstrained variance estimate                                                                                                              |
| lsdeg      | length of the vector sdegree. If lsdeg is not equal to 1 the user provided the vector of the unconditional variance estimator for the design points given by the vector N   |
| inverse    | indicates, if the origin variance function or its inverse has been estimated                                                                                                |
| estimation | the monotone estimate for the variance function at the design points $t$                                                                                                    |

**Author(s)**

This R Package was developed by Kay Pilz and Stefanie Titoff. Earlier developments of the estimator were made by Holger Dette and Kay Pilz.

**See Also**

monreg for monotone regression function estimation and monvardiff for monotone variance function estimation by differences.



# Index

\*Topic **nonparametric**

monreg, 2

monvardiff, 4

monvarresid, 6

\*Topic **regression**

monreg, 2

monvardiff, 4

monvarresid, 6

\*Topic **smooth**

monreg, 2

monvardiff, 4

monvarresid, 6

\*Topic

monvardiff, 4

monreg, 2

monvardiff, 4

monvarresid, 6