Package 'h0'

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Title A Meta Analysis for Estimating the Hubble Constant via Time
Delay Cosmography

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Description We provide a toolbox to conduct a meta analysis for estimating the current expansion rate of the Universe, called the Hubble constant H0, via time delay cosmography. The input data are (i) estimates of Fermat potential differences and their one-sigma uncertainties (standard errors), and (ii) estimates of time delays and their one-sigma uncertainties. Given these inputs, the meta analysis conducts a maximum likelihood estimation or a Bayesian inference to infer the Hubble constant.

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h0-package	*A Meta Analysis for Estimating the Hubble Constant via Time Delay Cosmography*

Description

The R package **h0** provides a toolbox to conduct a meta analysis for estimating the current expansion rate of the Universe, called the Hubble constant H0, via time delay cosmography. The input data are (i) estimates of Fermat potential differences and their one-sigma uncertainties (standard errors), and (ii) estimates of time delays and their one-sigma uncertainties. Given these inputs, the meta analysis conducts a maximum likelihood estimation or a Bayesian inference to infer the Hubble constant.

Details

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Author(s)

Hyungsuk Tak

References

T. Treu and P. Marshall (2016) <doi:10.1007/s00159-016-0096-8> "Time Delay Cosmography".

h0	*Obtaining Bayesian posterior sample or maximum likelihood estimate of the Hubble constant via a meta analysis*

Description

The function h0 conducts a meta analysis to infer the current expansion rate of the University (Hubble constant) via maximum likelihood estimation or Bayesian posterior sampling. The analysis is constructed under time delay cosmography. The minimum inputs to implement this function are (1) time delay estimates (TD.est); (2) their standard errors (TD.se); (3) Fermat potential difference estimates (FPD.est); (4) their standard errors (FPD.se); (5) redshifts of deflectors (lenses) (z.d); and (6) redshifts of sources (quasars) (z.s). We note that the length of each input vector must be

identical, and the order of input values in each vector must be consistent across all of the vectors. For example, let's say the third input value in one of the input vectors, `TD.est`, is a time delay estimate between gravitationally lensed images A and C of a specific quasar. Then the third input value in the other input vectors must be a quantity corresponding to the gravitationally lensed images A and C of that specific quasar.

Usage

```
h0(TD.est, TD.se,
    FPD.est, FPD.se,
    z.d, z.s,
    h0.bound = c(60, 80),
    h0.grid.size = 400,
    omega.bound = c(0.05, 0.5),
    omega.grid.size = 200,
    sample.size = 1e5,
    method = "mle")
```

Arguments

`TD.est`	A vector for time delay estimates. For example, if a data set contained three time delay estimates of a quad-lens system and one time delay estimate of a double-lens system, then this vector would be composed of the three time delay estimates from the quad and one time delay estimate from the double.
`TD.se`	A vector for standard errors (one-sigma uncertainties) of the time delay estimates. For example, if a data set contained three time delay estimates of a quad-lens system and one time delay estimate of a double-lens system, then this vector would be composed of the three standard errors corresponding to the first three time delay estimates from the quad and one standard error corresponding to the single time delay estimate from the double.
`FPD.est`	A vector for Fermat potential difference estimates. For example, if a data set contained three Fermat potential difference estimates of a quad-lens system and one Fermat potential difference estimate of a double-lens system, then this vector would be composed of the three Fermat potential difference estimates from the quad and one Fermat potential difference estimate from the double.
`FPD.se`	A vector for standard errors (one-sigma uncertainties) of the Fermat potential difference estimates. For example, if a data set contained three Fermat potential difference estimates of a quad-lens system and one Fermat potential difference estimate of a double-lens system, then this vector would be composed of the three standard errors corresponding to the first three Fermat potential difference estimates from the quad and one standard error corresponding to the single Fermat potential difference estimate from the double.
`z.d`	A vector for redshifts of deflectors (lenses). For example, if a data set contained three time delay estimates of a quad-lens system and one time delay estimate of a double-lens system, then users need to fill out the first three spots of this vector by the same redshift of the quad-lens, and fill out the last spot by the redshift of the double-lens.

z.s   A vector for redshifts of sources (quasars). For example, if a data set contained
               three time delay estimates of a quad-lens system and one time delay estimate of
               a double-lens system, then users need to fill out the first three spots of this vector
               by the same redshift of the quadruply lensed quasar, and fill out the last spot by
               the redshift of the doubly-lensed quasar.

h0.bound   It determins the search range for the Hubble constant for both maximum likeli-
               hood estimation and posterior sampling. Default is to search possible values of
               the Hubble constant between 60 and 80. Users can set a wider range, if needed.

h0.grid.size   Optional if a Bayesian inference is to be conducted with method = "bayes". It
               determins the number of values on a grid of the Hubble constant. Default is to
               set it to 400, meaning that the grid sampling method will start on a grid of 400
               values of the Hubble constant within the range (set by h0.bound). For more
               accurate grid sampling, it is recommended to increase this size when the search
               region increases.

omega.bound   It determins the search range for the current dark matter density for both maxi-
               mum likelihood estimation and posterior sampling. Default is to search possible
               values of the current dark matter density between 0.05 and 0.5. Users can set a
               wider range, if needed.

omega.grid.size
               Optional if a Bayesian inference is to be conducted with method = "bayes". It
               determins the number of values on a grid of the current dark matter density.
               Default is to set it to 200, meaning that the grid sampling method will start on
               a grid of 200 values of the current dark matter density within the range (set by
               omega.bound). For more accurate grid sampling, it is recommended to increase
               this size when the search region increases.

sample.size   Optional if a Bayesian inference is to be conducted with method = "bayes".
               It determines the posterior sample size for each of the two model parameters,
               the Hubble constant and the current dark matter density, to be drawn by a grid
               sampling method. Its default value is 100,000.

method   Maximum likelihood estimation is conducted if method = "mle", while Bayesian
               posterior sampling is done if method = "bayes".

Details

Time delay cosmography infers the Hubble constant H_0 by modeling time delays caused by strong
gravitational lensing. The following equation is the basis of time delay cosmography:

$$c\Delta_{ijk} = D_\Delta(H_0, z_k, \Omega)\phi_{ijk},$$

where c denotes the speed of light, and Δ_{ijk} is the time delay in days between lensed images i
and j of quasar k ($k = 1, 2, \ldots, n$). The notation $z_k = \{z_{sk}, z_{dk}\}$ indicates a vector for two
redshifts of the k-th source (z_{sk}) and deflector (z_{dk}). The next notation $\Omega = \{\Omega_m, \Omega_\Lambda\}$ indicates
a vector for the present-day dark matter density Ω_m and dark energy density Ω_Λ. Since their sum
becomes one under the flat ΛCDM model (standard cosmology), we treat Ω_m as the only unknown
parameter. The notation $D_\Delta(H_0, z_k, \Omega)$ denotes the time delay distance in the unit of megaparsec
(Mpc), which is a deterministic function of H_0, z_k, and Ω.

We emphasize that the physical equation deterministically relates five quantities, i.e., time delay Δ_{ijk}, the Hubble constant H_0, redshifts z_k, cosmological parameter Ω, and Fermat potential difference ϕ_{ijk}. That means, if we completely knew four quantities, Δ_{ijk}, z_k, Ω, and ϕ_{ijk}, then H_0 can be easily determined to be a single number. The meta analysis takes advantage of this relationship, marginalizes nuisance parameters, and returns estimates of H_0 and Ω via maximum likelihood estimation or Bayesian posterior sampling.

Value

The outcomes of `h0` are composed of:

h0.mle A constant. The maximum likelihood estimate of the Hubble constant, available when `method = "mle"`.

h0.se A constant. An asymptotic one-sigma uncertainty (standard error) of the maximum likelihood estimate of the Hubble constant, available when `method = "mle"`. This is computed from the curvature of the likelihood function evaluated at the maximum likelihood estimate.

omega.mle A constant. The maximum likelihood estimate of the current dark matter density, available when `method = "mle"`.

omega.se A constant. An asymptotic one-sigma uncertainty (standard error) of the current dark matter density estimate of the Hubble constant, available when `method = "mle"`. This is computed from the curvature of the likelihood function evaluated at the maximum likelihood estimate.

h0 A vector. The posterior sample of the Hubble constant, available when `method = "bayes"`. Its sample size is set by users.

omega A vector. The posterior sample of the current dark matter density, available when `method = "bayes"`. Its sample size is set by users.

contour A matrix. All of the likelihood values computed on the two-dimensional grid, available when `method = "bayes"`. This matrix can be used to produce a contour plot

h0.grid A vector. It contains the grid of values of the Hubble constant that has been used during the grid sampling procedure, available when `method = "bayes"`. This can also be used to draw a contour plot.

omega.grid A vector. It contains the grid of values of the current dark matter density that has been used during the grid sampling procedure, available when `method = "bayes"`. This can also be used to draw a contour plot.

Author(s)

Hyungsuk Tak

References

T. Treu and P. Marshall (2016) <doi:10.1007/s00159-016-0096-8> "Time Delay Cosmography".

Examples

```
#####  Suppose we have three time delay estimates of a quad-lens system,
```

```
#####  and one time delay estimate of a double-lens system.
time.delay.est <- c(-101, -91.7, -104.2, -18.8)

#####  Each time delay estimate accompanies its one-sigma uncertainty (standard error)
time.delay.se <- c(4.4, 4.2, 4.1, 0.9)

#####  The redshift of the deflector in the quad-lens system is 0.868,
#####  and that in the double-lens system is 0.512.
z.d.obs <- c(0.868, 0.868, 0.868, 0.512)

#####  The redshift of the source in the quad-lens system is 2.621,
#####  and that in the double-lens system is 3.37.
z.s.obs <- c(2.621, 2.621, 2.621, 3.37)

#####  Three Fermat potential difference estimates are obtained from a quad-lens syatem,
#####  and one estimate is from the double-lens system.
fermat.diff.est <- c(-0.6395, -0.5734, -0.6431, -0.2625)

#####  Each Fermat potential difference estimate accompanies
#####  its one-sigma uncertainty (standard error).
fermat.diff.se <- c(0.0265, 0.0254, 0.0299, 0.0148)

#### Next, we make sure that each vector has the same length.
length(z.d.obs); length(z.s.obs); length(fermat.diff.est);
length(fermat.diff.se); length(time.delay.est); length(time.delay.se)

#####  The following is an example code to obtain 1000 posterior samples of the Hubble constant
#####  and those of the current dark matter density.
#####  In practice, users may want to increase the grid sizes for a more accuracte inference,
#####  even though it increases computational burden.
res <- h0(TD.est = time.delay.est, TD.se = time.delay.se,
          FPD.est = fermat.diff.est, FPD.se = fermat.diff.se,
          z.d = z.d.obs, z.s = z.s.obs,
          h0.bound = c(60, 80), h0.grid.size = 50,
          omega.bound = c(0.05, 0.5), omega.grid.size = 25,
          sample.size = 1000, method = "bayes")

#####  We can draw a contour plot, and the following two histograms,
#####  using the output from the Bayesian fit.
contour(res$h0.grid, res$omega.grid, res$contour,
        nlevels = 5, drawlabels = FALSE,
        ylim = c(0.05, 0.5), xlim = c(60, 80), levels = c(0.01, 0.25, 0.5, 0.75, 0.99))

hist(res$h0, 20, prob = TRUE, col = 0, xlim = c(60, 80))

hist(res$omega, 30, prob = TRUE, col = 0, xlim = c(0.05, 0.55))

#####  The following is an example code to obtain the maximum likelihood estimate of
#####  the Hubble constant and its asymptotic uncertainty.
res.mle <- h0(TD.est = time.delay.est, TD.se = time.delay.se,
              FPD.est = fermat.diff.est, FPD.se = fermat.diff.se,
              z.d = z.d.obs, z.s = z.s.obs,
              h0.bound = c(60, 80), omega.bound = c(0.05, 0.5),
```

```
                method = "mle")
```

```
res.mle$h0.mle; res.mle$h0.se; res.mle$omega.mle; res.mle$omega.se
```

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