Package 'Barycenter'

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Type Package

Title Regularized Wasserstein Distances and Barycenters

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Description Computations of entropy regularized Wasserstein Distances, a.k.a. dual-Sinkhorn divergences, and entropy regularized Wasserstein Barycenters. Relevant papers are Marco Cuturi (2013) <arXiv:1306.0895>, Marco Cuturi (2014) <arXiv:1310.4375> and Jason Altschuler et al. <arXiv:1705.09634>.

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eight *MNIST dataset of the digit eight (small)*

Description

A list of five images of the digit eight. Each image is a `28 x 28` matrix, representing the weights of the corresponding pixels scaled s.t. they sum up to one. The images are included in a list. This dataset is based on the MNIST dataset.

Usage

```
eight
```

Format

List

Examples

```
#Take a look into the dataset, e.g. the first image of the digit eight.
eight[[1]]
image(eight[[1]])
```

Greenkhorn *Greenkhorn Distances (approximation to EMD)*

Description

The `Greenkhorn` algorithm to approximate the earth movers distance (EMD), a.k.a. Wasserstein distance, between two probability vectors `r` and `c` with specified cost-matrix `costm`.

Usage

```
Greenkhorn(r, c, costm, lambda = 1, maxIter = 10000, tolerance=10^(-8))
```

Arguments

`r`	(n x 1) row vector in the probability simplex (nonnegative summing to one).
`c`	(1 x m) row vector in the probability simplex (nonnegative summing to one).
`costm`	(n x m) matrix of pairwise distances/costs between bins with mass described by `r` and bins with mass described by `c`.
`lambda`	Non-negative regularization parameter (for small `lambda` the Sinkhorn Distance is close to the EMD).
`maxIter`	Maximum number of iterations.
`tolerance`	A threshold for the integrated stopping criterion based on marginal differences.

Value

Returns a list containing the regularized transport plan represented as a nxm matrix as well as the Sinkhorn distance between the given marginals `r` and `c`.

Author(s)

Marcel Klatt

References

Altschuler, J., Weed, J. and Rigollet, P.: `Near-linear time approximation algorithms for optimal transport via S`
Advances in Neural Information Processing Systems 30 (NIPS 2017)

Examples

```
#Sinkhorn Distances between the first image to the second image in the dataset eight.
#We creat costm simply using a distance matrix on the grid [0,1]x[0,1].
n <- seq(0,1,length.out = dim(eight[[1]])[2])
costm <- as.matrix(dist(expand.grid(n,rev(n)), diag=TRUE, upper=TRUE))
r <- matrix(eight[[1]],28*28,1)
c <- matrix(eight[[2]],1,28*28)
Greenkhorn(r, c, costm)$Distance
```

`Sinkhorn` *Sinkhorn Distances (upper bound to EMD)*

Description

The `Sinkhorn` algorithm to compute N dual-Sinkhorn divergences, i.e. upper bounds on the earth movers distance (EMD), a.k.a. Wasserstein distance.

Usage

```
Sinkhorn(a, b, costm, lambda = 1, maxIter = 10000, tolerance=10^(-8))
```

Arguments

`a`	Either a d_1x1 column vector in the probability simplex (nonnegative summing to one) or a d_1xN matrix, where each column is in the probability simplex.
`b`	A d_1xN matrix of N vectors in the probability simplex.
`costm`	A matrix of pairwise distances/costs between bins described in `a` and bins in the b_1,...,b_N histograms.
`lambda`	Non-negative regularization parameter (for small `lambda` the Sinkhorn Distance is close to the EMD).
`maxIter`	Maximum number of iterations.
`tolerance`	A threshold for the integrated stopping criterion based on marginal differences.

Value

Returns the Sinkhorn Distances between the given bins. If a is given by a d_1x1 column vector the function returns the distances

$$[D(a, b_1), ..., D(a, b_N)].$$

If a is given by a d_1xN matrix the function returns the distances

$$[D(a_1, b_1), ..., D(a_N, b_N)].$$

If a and b are simply given by two d_1x1 and d_2x1 vectors each in the probability simplex, respectively, then the functions returns a list containing the Sinkhorn Distance as well as the corresponding regularized transport plan.

Author(s)

Marcel Klatt

References

Cuturi, M.: Sinkhorn Distances: Lightspeed Computation of Optimal Transport, Advances in Neural Information Processing Systems 26, 2013

Examples

```
#Sinkhorn Distances between the first image to the remaining four images in the dataset eight.
#We creat costm simply using a distance matrix on the grid [0,1]x[0,1].
n <- seq(0,1,length.out = dim(eight[[1]])[2])
costm <- as.matrix(dist(expand.grid(n,rev(n)), diag=TRUE, upper=TRUE))
a <- matrix(eight[[1]],28*28,1)
b <- matrix(c(eight[[2]],eight[[3]],eight[[4]],eight[[5]]),28*28,4)
Sinkhorn(a, b, costm)
```

---

three *MNIST dataset of the digit three*

Description

A list of 4351 images of the digit three. Each image is a 28 x 28 matrix, representing the weights of the corresponding pixels scaled s.t. they sum up to one. The images are included in a large list. This dataset is based on the MNIST dataset.

Usage

```
three
```

Format

List

Examples

```
#Take a look into the dataset, for example the 25th image of the digit three.
three[[25]]
image(three[[25]])
```

`WaBarycenter` *Regularized Wasserstein Barycenters*

Description

`WaBarycenter` takes in a list of matrices representing joint measures on the row and column space and outputs the corresponding Barycenter. The list has to consist of matrices having all the same dimensions, for instance, each matrix represents the normalized weights of the corresponding pixels of images.

Usage

```
WaBarycenter(images, maxIter = 10, lambda = FALSE, costm = FALSE)
```

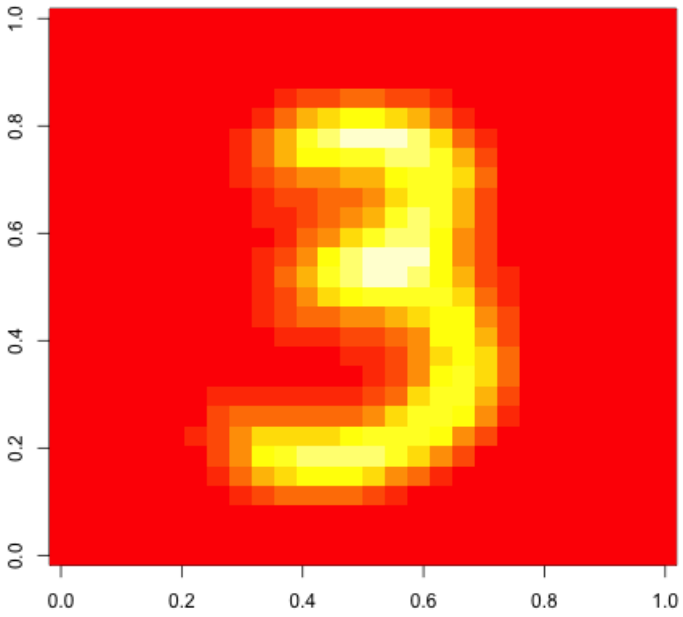
Arguments

`images`	A list of matrices satisfying the prerequisites described above.
`maxIter`	Maximum number of iterations.
`lambda`	Non-negative regularization parameter (for large lambda the regularized Barycenter is close to its true counterpart). If FALSE the algorithm uses a lambda depending on `costm`.
`costm`	A matrix of pairwise distances between the locations. If FALSE the algorithm uses the usual euclidean distance matrix on a [0,1]x[0,1] equidistant pixel grid.

Value

The Barycenter of the matrices, represented by a nxm matrix.

Given the MNIST dataset, a Barycenter of the digit three is shown below. The Barycenter is based on 4351 images each represented by a 28 x 28 pixel grid, respectively. The values for `lambda` and `maxIter` were set by default. The dataset is also available in this package (c.f. three).



Author(s)

Marcel Klatt

References

Cuturi, M.: Fast Computation of Wasserstein Barycenters, Proceedings of the International Conference on Machine Learning, Beijing, China, 2014

Examples

```
#Computation of a Barycenter based on five images representing the digit eight, respectively.
WaBarycenter(eight,lambda=10)
#For a more reasonable but longer computation!
## Not run: WaBarycenter(eight)
```

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